Chapter 2

Modeling in the frequency domain

Solutions

CONTROL SYSTEMS ENGINEERING

Sixth Edition, Norman S. Nise

8.

a. Cross multiplying, $(s^2+5s+10)X(s) = 7F(s)$.

Taking the inverse Laplace transform, $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 10x = 7f$.

b. Cross multiplying after expanding the denominator, $(s^2+21s+110)X(s) = 15F(s)$.

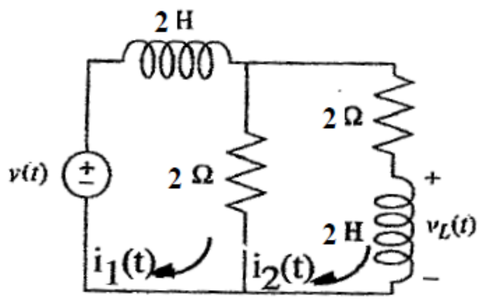
Taking the inverse Laplace transform, $\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 110x = 15f$.

c. Cross multiplying, $(s^3+11s^2+12s+18)X(s) = (s+3)F(s)$.

Taking the inverse Laplace transform, $\frac{d^3x}{dt^3} + 11\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 18x = df/dt + 3f$.

17.

a.



Writing mesh equations

$$(2s+2)I_1(s) - 2I_2(s) = V_i(s)$$

$$-2I_1(s) + (2s+4)I_2(s) = 0$$

But from the second equation, $I_1(s) = (s+2)I_2(s)$. Substituting this in the first equation yields,

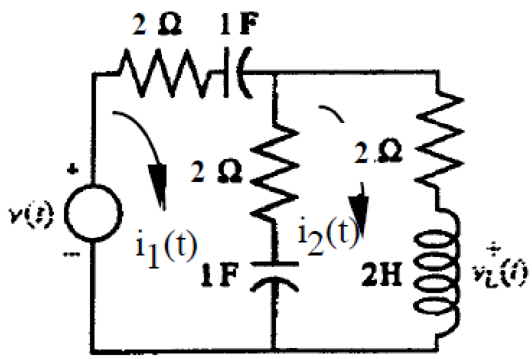
$$(2s+2)(s+2)I_2(s) - 2\,I_2(s) = V_i(s)$$

or

$$I_2(s)/V_i(s) = 1/(2s^2 + 4s + 2)$$

But, $V_L(s) = sI_2(s)$. Therefore, $V_L(s)/V_i(s) = s/(2s^2 + 4s + 2)$.

b.



$$(4+\frac{2}{s})I_1(s)-(2+\frac{1}{s})I_2(s)=V(s)$$

$$-(2+\frac{1}{s})I_1(s)+(4+\frac{1}{s}+2s)=0$$

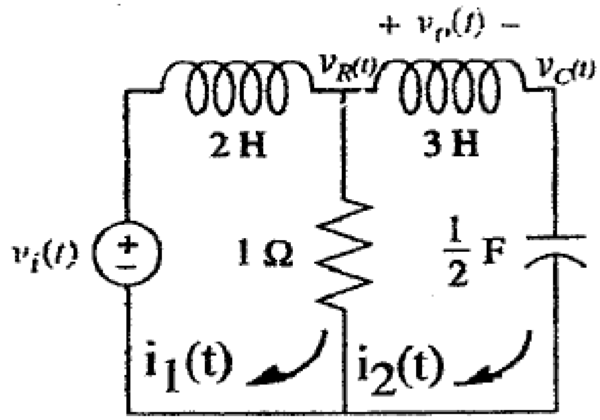
Solving for $I_2(s)$:

$$I_2(s)=\frac{\begin{vmatrix}\frac{4s+2}{s} & V(s)\\ \frac{-(2s+1)}{s} & 0\end{vmatrix}}{\begin{vmatrix}\frac{4s+2}{s} & \frac{-(2s+1)}{s}\\ \frac{-(2s+1)}{s} & \frac{(2s^2+4s+1)}{s}\end{vmatrix}}=\frac{sV(s)}{4s^2+6s+1}$$

Therefore, $$\frac{V_L(s)}{V(s)}=\frac{2sI_2(s)}{V(s)}=\frac{2s^2}{4s^2+6s+1}$$

18.

a.



Writing mesh equations,

$$(2s + 1)I_1(s) - I_2(s) = V_i(s)$$

$$-I_1(s) + (3s + 1 + 2/s)I_2(s) = 0$$

Solving for $I_2(s)$,

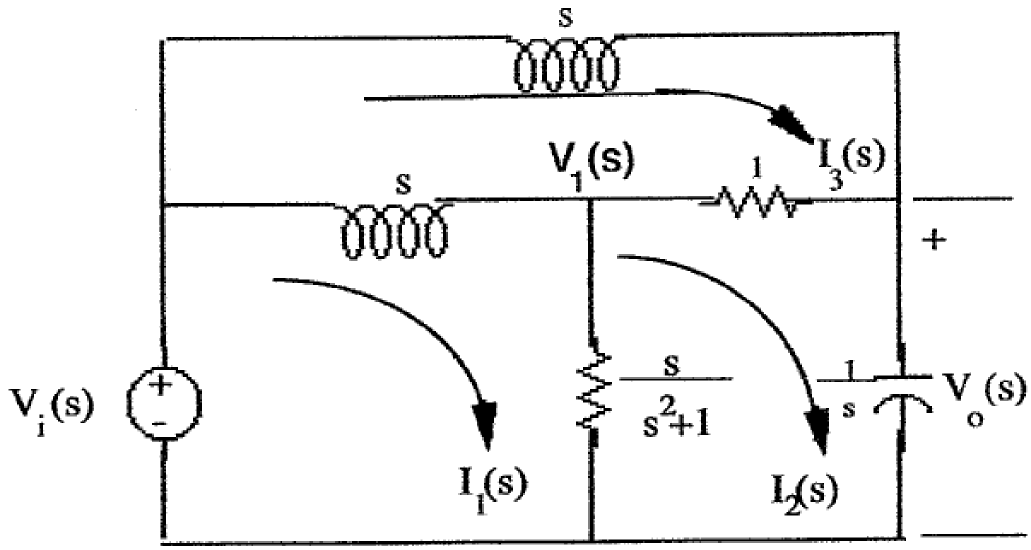
$$I_2(s) = \frac{\begin{vmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2s+1 & -1 \\ -1 & \dfrac{3s^2+s+2}{s} \end{vmatrix}}$$

Solving for $I_2(s)/V_i(s)$,

$$\frac{I_2(s)}{V_i(s)} = \frac{s}{6s^3 + 5s^2 + 4s + 2}$$

But $V_o(s) = I_2(s)3s$. Therefore , $G(s) = 3s^2/(6s^3 + 5s^2 + 4s + 2)$.

b. Transforming the network yields,



Writing the loop equations,

$$(s+\frac{s}{s^2+1})I_1(s)-\frac{s}{s^2+1}I_2(s)-sI_3(s)=V_i(s)$$

$$-\frac{s}{s^2+1}I_1(s)+(\frac{s}{s^2+1}+1+\frac{1}{s})I_2(s)-I_3(s)=0$$

$$-sI_1(s)-I_2(s)+(2s+1)I_3(s)=0$$

Solving for $I_2(s)$,

$$I_2(s)=\frac{s(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}V_i(s)$$

But, $V_o(s)=\frac{I_2(s)}{s}=\frac{(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}V_i(s)$. Therefore,

$$\frac{V_o(s)}{V_i(s)}=\frac{s^2+2s+2}{s^4+2s^3+3s^2+3s+2}$$

25.

Let $X_1(s)$ be the displacement of the left member of the spring and $X_3(s)$ be the displacement of the mass.

Writing the equations of motion

$$2x_1(s) - 2x_2(s) = F(s)$$
$$-2X_1(s) + (5s+2)X_2(s) - 5sX_3(s) = 0$$
$$-5sX_2(s) + (10s^2 + 7s)X_3(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} 5s^2+10 & F(s) \\ -10 & 0 \end{vmatrix}}{\begin{vmatrix} 5s^2+10 & -10 \\ -10 & \frac{1}{5}s+10 \end{vmatrix}} = \frac{10F(s)}{s(s^2+50s+2)}$$

Thus, $\dfrac{X_2(s)}{F(s)} = \dfrac{1}{10}\dfrac{(10s+7)}{s(5s+1)}$

29.

Writing the equations of motion,

$$(4s^2+4s+8)X_1(s) - 4X_2(s) - 2sX_3(s) = 0$$
$$-4X_1(s) + (5s^2+3s+4)X_2(s) - 3sX_3(s) = F(s)$$
$$-2sX_1(s) - 3sX_2(s) + (5s^2+5s+5) = 0$$

30.

a.

Writing the equations of motion,

$$(5s^2+9s+9)\theta_1(s) - (s+9)\theta_2(s) = 0$$
$$-(s+9)\theta_1(s) + (3s^2+s+12)\theta_2(s) = T(s)$$

b.

Defining

$\theta_1(s)$ = rotation of J_1

$\theta_2(s)$ = rotation between K_1 and D_1

$\theta_3(s)$ = rotation of J_3

$\theta_4(s)$ = rotation of right - hand side of K_2

the equations of motion are

$$(J_1 s^2 + K_1)\theta_1(s) - K_1\theta_2(s) = T(s)$$

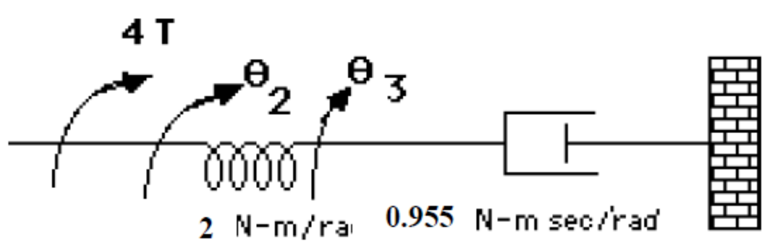
$$-K_1\theta_1(s) + (D_1 s + K_1)\theta_2(s) - D_1 s\,\theta_3(s) = 0$$

$$-D_1 s\,\theta_2(s) + (J_2 s^2 + D_1 s + K_2)\theta_3(s) - K_2\theta_4(s) = 0$$

$$-K_2\theta_3(s) + (D_2 s + (K_2 + K_3))\theta_4(s) = 0$$

35.

Reflecting impedances and applied torque to respective sides of the spring yields the following equivalent circuit:



Writing the equations of motion,

$$2\theta_2(s) - 2\,\theta_3(s) = 4.231T(s)$$

$$-2\theta_2(s) + (0.955s+2)\theta_3(s) = 0$$

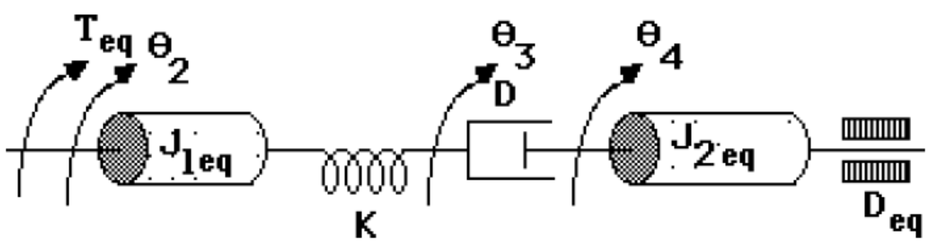
Solving for $\theta_3(s)$,

$$\theta_3(s) = \frac{\begin{vmatrix} 2 & 4.231T(s) \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & -2 \\ -2 & (0.955s+2) \end{vmatrix}} = \frac{8.462T(s)}{1.91s} = \frac{4.43T(s)}{s}$$

Hence, $\frac{\theta_3(s)}{T(s)} = \frac{4.43}{s}$. But, $\theta_4(s) = 0.192\theta_3(s)$. Thus, $\frac{\theta_4(s)}{T(s)} = \frac{0.851}{s}$.

37.

Reflect all impedances on the right to the viscous damper and reflect all impedances and torques on the left to the spring and obtain the following equivalent circuit:



Writing the equations of motion,

$$(J_{1eq}s^2+K)\theta_2(s) - K\theta_3(s) = T_{eq}(s)$$

$$-K\theta_2(s)+(Ds+K)\theta_3(s) - Ds\theta_4(s) = 0$$

$$-Ds\theta_3(s) + [J_{2eq}s^2 + (D+D_{eq})s]\theta_4(s) = 0$$

where: $J_{1eq} = J_2+(J_a+J_1)\left(\frac{N_2}{N_1}\right)^2$; $J_{2eq} = J_3+(J_L+J_4)\left(\frac{N_3}{N_4}\right)^2$; $D_{eq} = D_L\left(\frac{N_3}{N_4}\right)^2$; $\theta_2(s) = \theta_1(s)$

$\frac{N_1}{N_2}$.

43.

The parameters are:

$$\frac{K_t}{R_a}=\frac{T_s}{E_a}=\frac{5}{5}=1;\ K_b=\frac{E_a}{\omega}=\frac{5}{\frac{600}{\pi}2\pi\frac{1}{60}}=\frac{1}{4};\ J_m=16\left(\frac{1}{4}\right)^2+4\left(\frac{1}{2}\right)^2+1=3;\ D_m=32\left(\frac{1}{4}\right)^2=2$$

Thus,

$$\frac{\theta_m(s)}{E_a(s)}=\frac{\frac{1}{3}}{s(s+\frac{1}{3}(2+(1)(\frac{1}{4})))}=\frac{\frac{1}{3}}{s(s+0.75)}$$

Since $\theta_2(s)=\frac{1}{4}\,\theta_m(s)$,

$$\frac{\theta_2(s)}{E_a(s)}=\frac{\frac{1}{12}}{s(s+0.75)}.$$